3-4 Videos Guide

3-4a

- The Comparison Test:
  Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms.
  (i) If $\sum b_n$ is convergent and $a_n \leq b_n$ for all n, then $\sum a_n$ is also convergent.
  (ii) If $\sum b_n$ is divergent and $b_n \leq a_n$ for all n, then $\sum a_n$ is also divergent.

Exercise:

- Determine whether the series converges or diverges.
  $$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$$

3-4b

- The Limit Comparison Test:
  Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms. If $\lim_{n\to\infty} \frac{a_n}{b_n} = c$, where c is a positive, finite number, then either both series converge or both series diverge.
- Two special cases:
  - If $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ AND if $\sum b_n$ converges, then $\sum a_n$ also converges.
  - If $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$ AND if $\sum b_n$ diverges, then $\sum a_n$ also diverges.

Exercises:

Determine whether the series converges or diverges.

3-4c

- $\sum_{n=3}^{\infty} \frac{n+2}{(n+1)^3}$

3-4d

- $\sum_{n=1}^{\infty} \frac{6^n}{5^n-1}$
- $\sum_{k=1}^{\infty} \frac{k \sin^2 k}{1+k^3}$
- $\sum_{n=1}^{\infty} \frac{2}{\sqrt{n}+2}$